

Skyrmions in 2D & 3D Magnets with Broken Bulk & Surface Inversion Symmetry

Mohit Randeria
Ohio State University



Skyrmionics
Santa Fe, August 2017





**James
Rowland**

* Banerjee, Erten & MR,
Nature Physics **9**, 626 (2013)



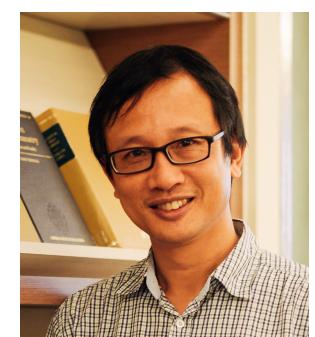
Po-Kuan Wu



**Sumilan
Banerjee**
(→ Bangalore)



Onur Erten
(→ Arizona State)



Ying-Jer Kao
National Taiwan U.

* Banerjee, Rowland, Erten & MR,
Phys. Rev. X **4**, 031045 (2014)

* Rowland, Banerjee & MR,
Phys. Rev. B **93**, 020404(R) (2016)

* Wu, Rowland, Kao & MR [unpublished]

* Ahmed, Rowland, Dunsiger, Esser,
McComb, MR & Kawakami;
arXiv:1706.08248



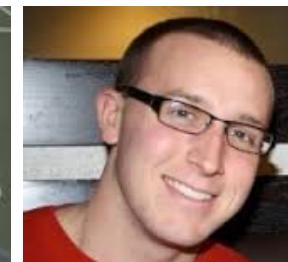
Adam Ahmed



Roland Kawakami



Sarah Dunsiger



Bryan Esser

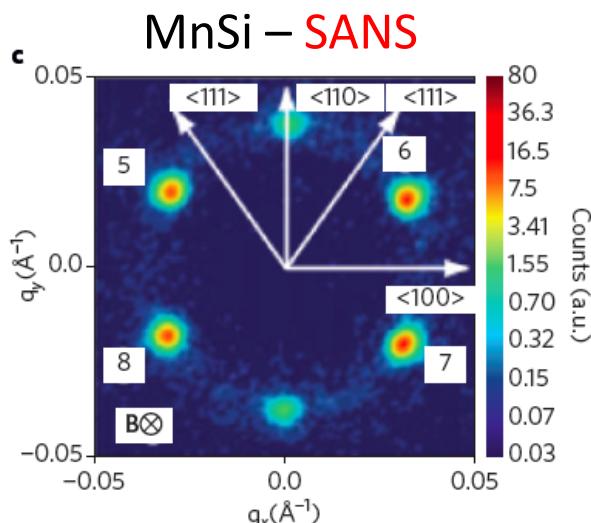


David McComb

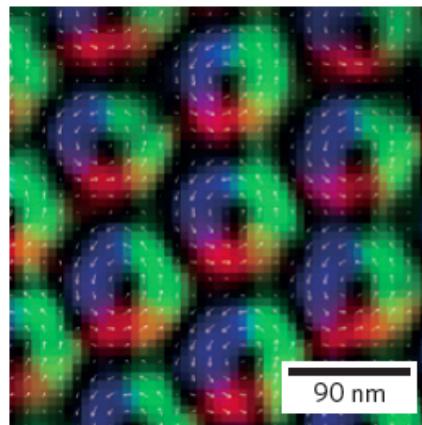
Outline:

- Introduction
- Skyrmions in 2D vs. 3D
 - * T=0 phase diagram & role of anisotropy
- Finite temperature Monte Carlo
 - * (T,H) phase diagram in 2D
- Rashba & Dresselhaus DMI
 - * Broken bulk vs. surface/mirror inversion
- Conclusions

Skyrmions: topological spin textures in chiral magnets



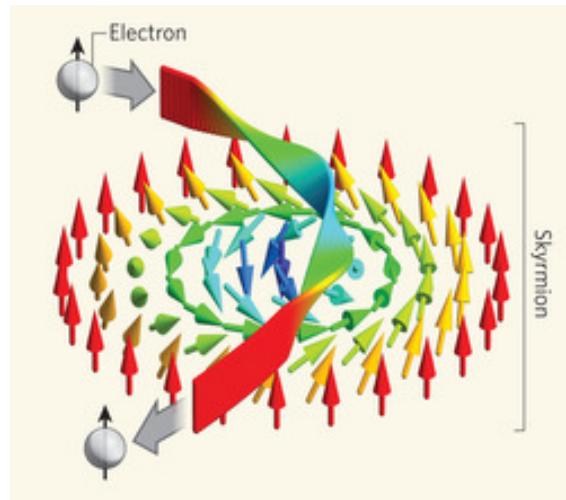
FeCoSi – Lorentz TEM



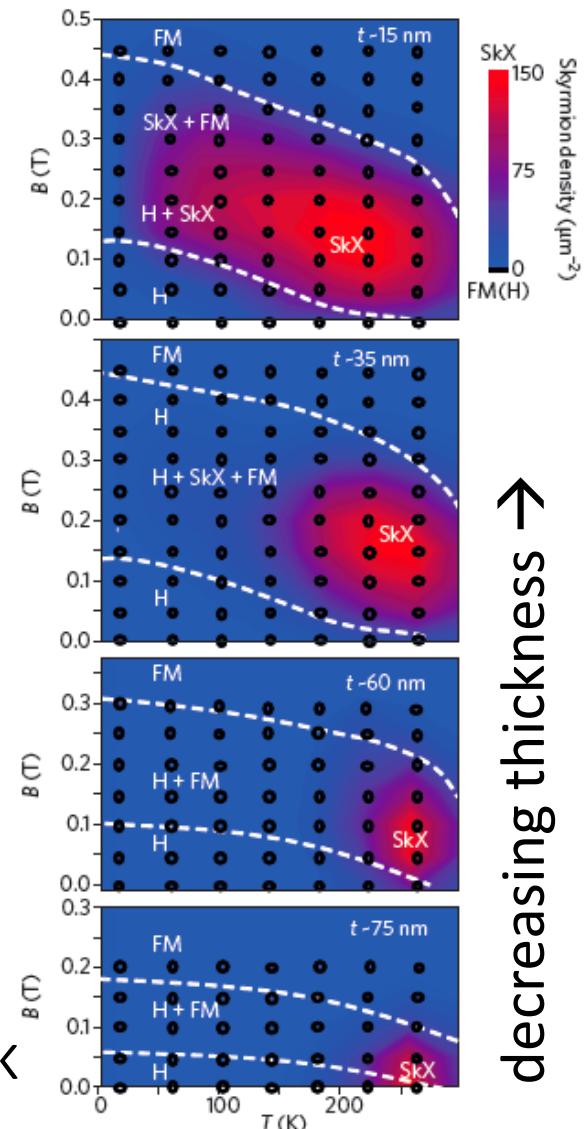
X. Z. Yu et al.,
Nature 465, 901 (2010)

Mühlbauer et al, Science 323, 915 (2009)

Berry Phase
→ Topological
Hall Effect



Pfleiderer & Rosch.
Nature, 465, 880 (2010)



FeGe
 $T_c = 278\text{K}$

Z. Yu et. al.,
Nature Mat 10, 106 (2011)

Chiral Magnetic Materials

- Ferromagnetic Exchange $-J \mathbf{S}_i \cdot \mathbf{S}_j$
- Chiral DM interaction (Dzyaloshinskii-Moriya)

$$\mathbf{D} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

Material constraints for DM:

1. Broken Inversion Symmetry → direction of D
2. Spin-orbit coupling (SOC) → magnitude of D

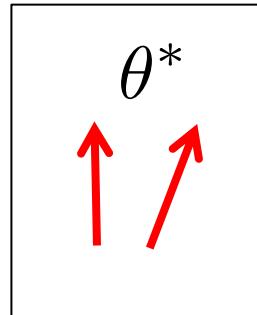
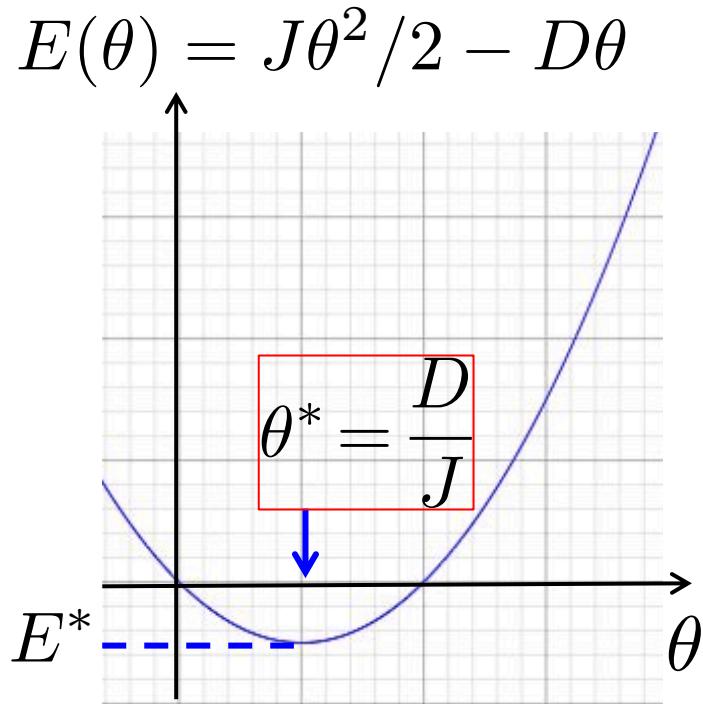
Broken Bulk Inversion: Non-centrosymmetric crystals (MnSi , FeGe , Cu_2OSeO_3 , ...)

Broken Mirror symmetry: polar crystals (GaV_4S_8 , ...)

Broken Surface inversion: Magnetic multilayers, interfaces & thin films

FM exchange + DMI \rightarrow Spin textures

- Ferromagnetic Exchange $-J \mathbf{S}_i \cdot \mathbf{S}_j$
 $\sim -J \cos \theta \sim J\theta^2$
- DM interaction $\mathbf{D} \cdot (\mathbf{S}_i \times \mathbf{S}_j) \simeq D \sin \theta \simeq D\theta$



Spontaneous
spin textures
Spirals & Skyrmions

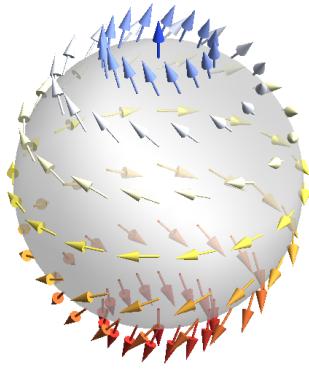
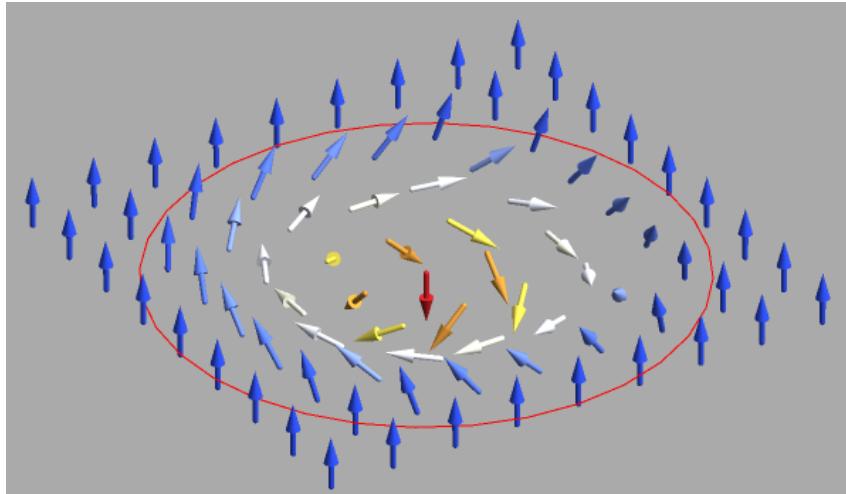
Length scale $\sim \left(\frac{J}{D} \right) a \gg a$
 $\sim 1 - 10^3 \text{ nm}$

Energy scales

$$\begin{cases} T_c \sim J \\ E^* = D^2/J \sim H_c \end{cases}$$

Skyrmion:

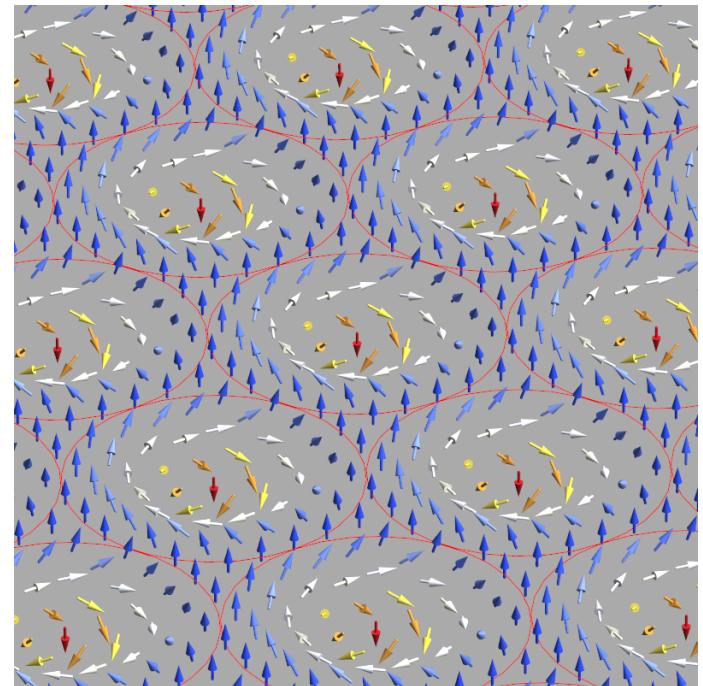
Topological spin texture in magnetization $\mathbf{M}(\mathbf{r}) = M \hat{\mathbf{m}}(\mathbf{r})$



“Winding Number” on unit sphere in spin-space

$$N_{\text{sk}} = \frac{1}{4\pi} \int d^2\mathbf{r} \ \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}}) = 0, \pm 1, \pm 2, \dots$$

Skyrmion Crystal (SkX)



Topological
Invariant

$$\pi_2(S^2) = \mathbb{Z}$$

Outline:

- Introduction
- Skyrmions in 2D vs. 3D
 - * T=0 phase diagram & role of anisotropy
- Finite temperature Monte Carlo
 - * (T,H) phase diagram in 2D
- Rashba & Dresselhaus DMI
 - * Broken bulk vs. surface/mirror inversion
- Conclusions

Phase Diagram of Chiral Magnets

- $T = 0 \rightarrow$ Minimize Energy

- * FM $\rightarrow J$

- * DMI $\rightarrow D$

- * Anisotropy $\rightarrow +K m_z^2$

- * Field $\rightarrow -H m_z$

- * **3D system:** Bulk samples

- * **2D systems:** thickness $\ll J/D$ ($a = 1$)

Choosing scales for
Length $(J/D)a$
& Energy D^2/J

→ Two dimensionless
parameters

KJ/D^2 and HJ/D^2

Continuum Field Theory (discretized for numerics)

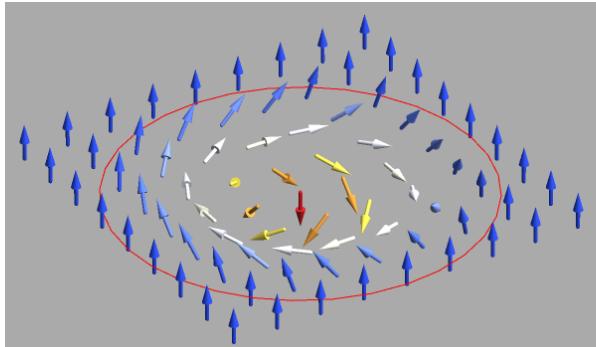
- * Analytical Variational calculation at $T=0$

- * Numerical Conjugate gradient minimization at $T=0$

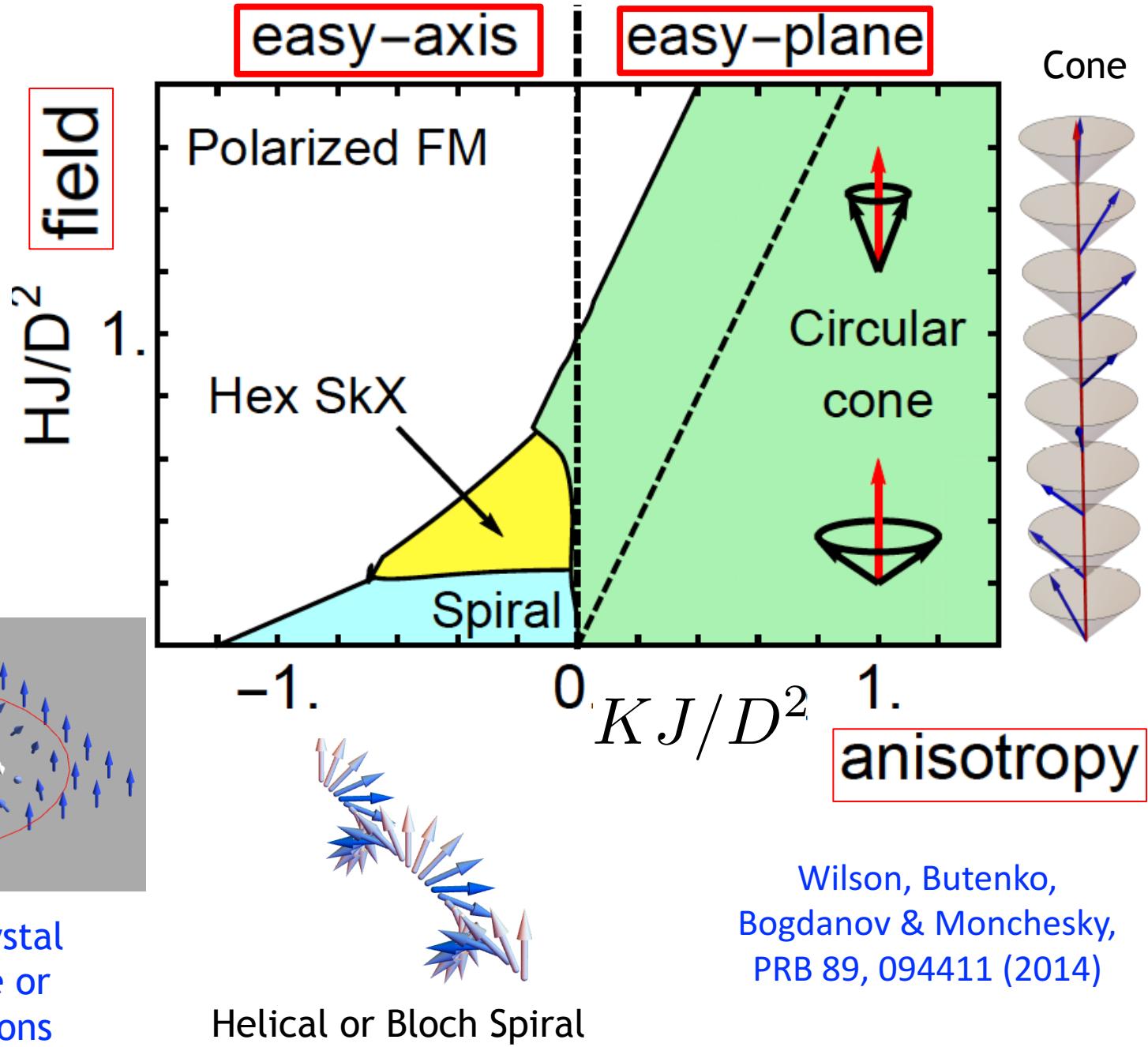
- * $T > 0$ Monte Carlo (later in talk)

3D Phase Diagram with Broken Bulk Inversion

$T=0$



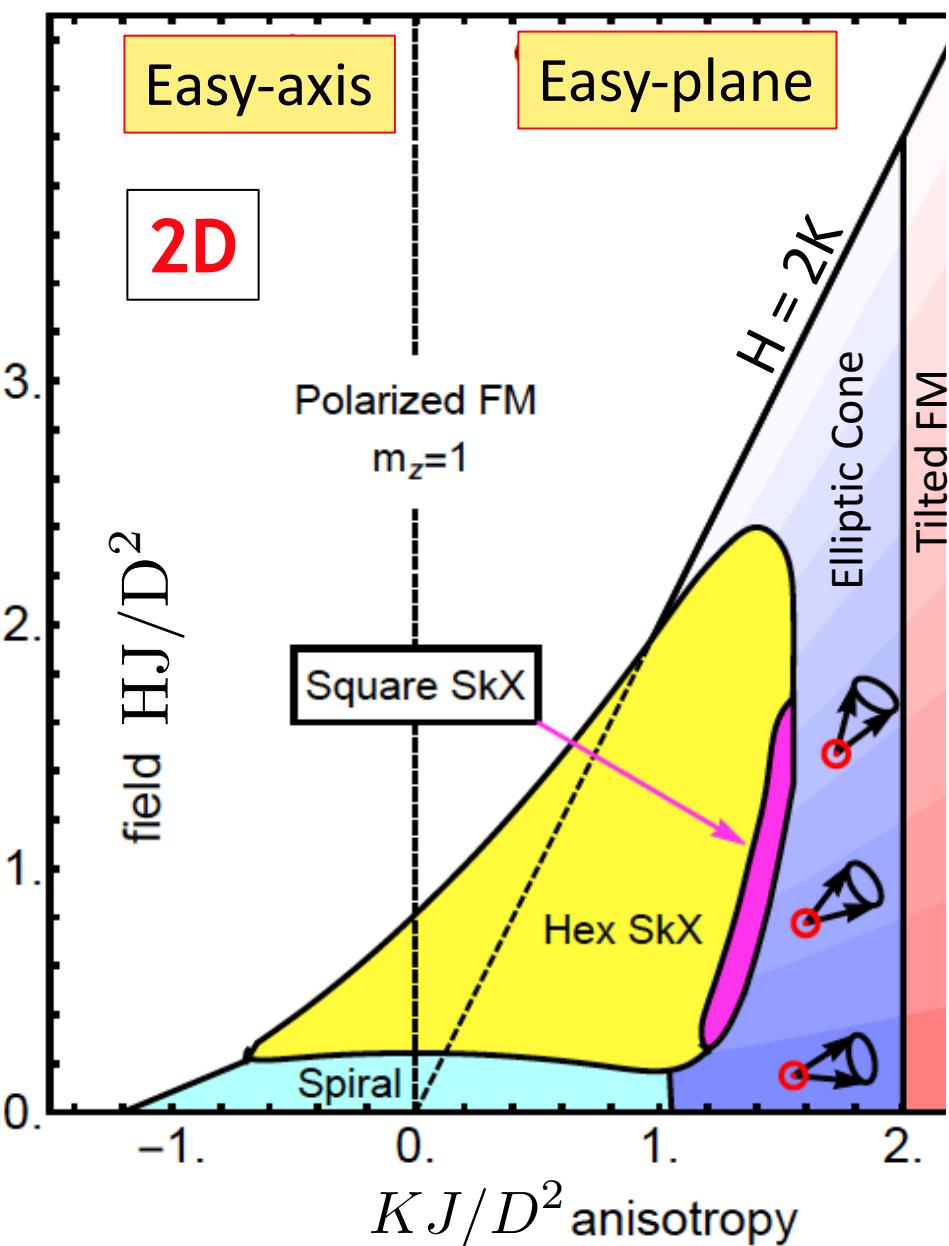
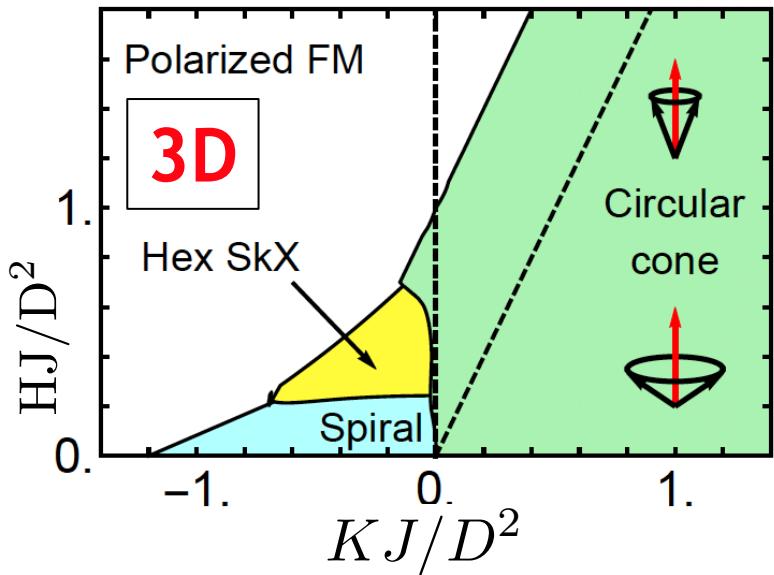
Hexagonal Crystal
of Vortex-like or
Bloch skyrmions



2D vs. 3D Phase Diagram

In 2D

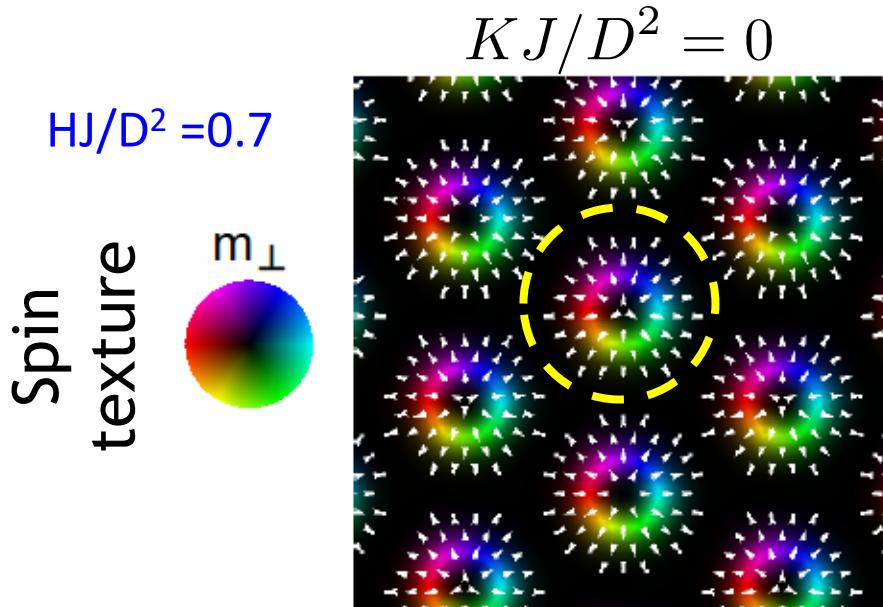
- * Skyrmion phase greatly enhanced
- * Importance of easy-plane anisotropy
- * No Cone phase



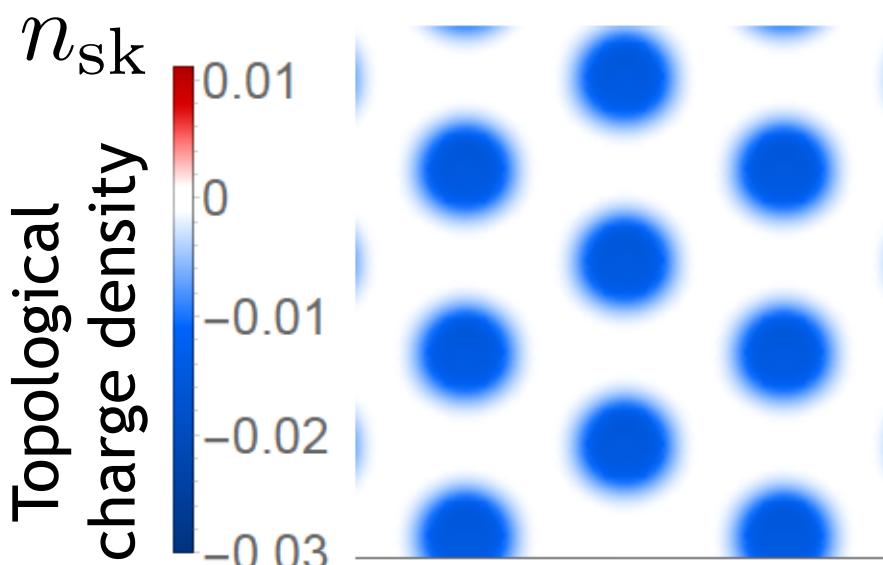
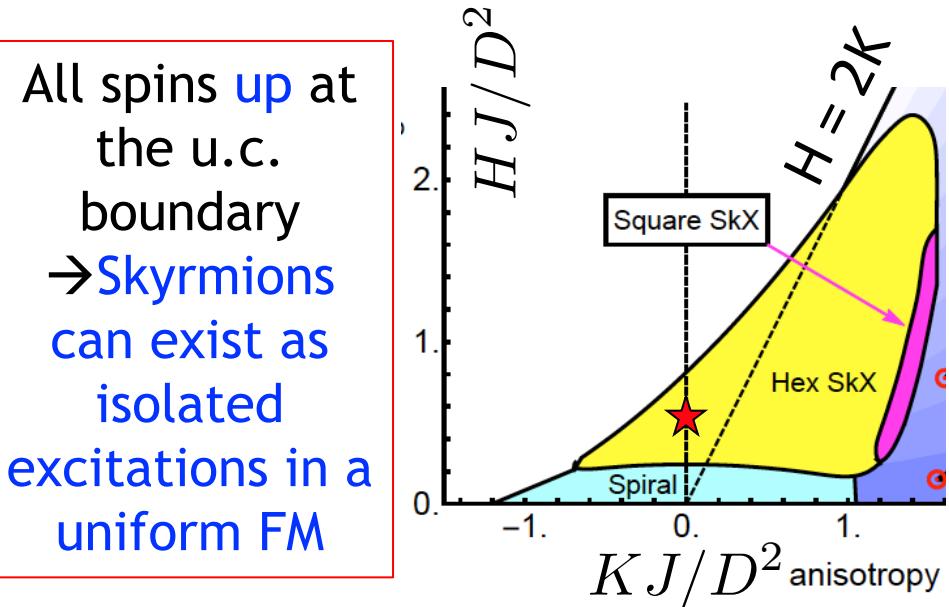
Banerjee, Rowland, Erten & MR, PRX (2014)
Rowland, Banerjee & MR, PRB (2016)

Square Lattice, see also:
Lin, Saxena & Batista, (2015)

Textures & Topological Charge in 2D: $H > 2K$



All spins **up** at
the u.c.
boundary
→ **Skyrmions**
can exist as
isolated
excitations in a
uniform FM



$$n_{\text{sk}}(\mathbf{r}) = \frac{1}{4\pi} \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}})$$

← $N_{\text{sk}} = \int d^2 \mathbf{r} n_{\text{sk}}(\mathbf{r}) = -1$

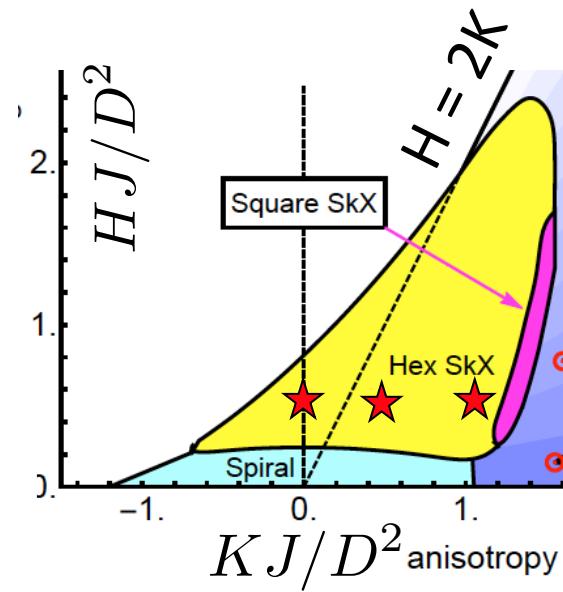
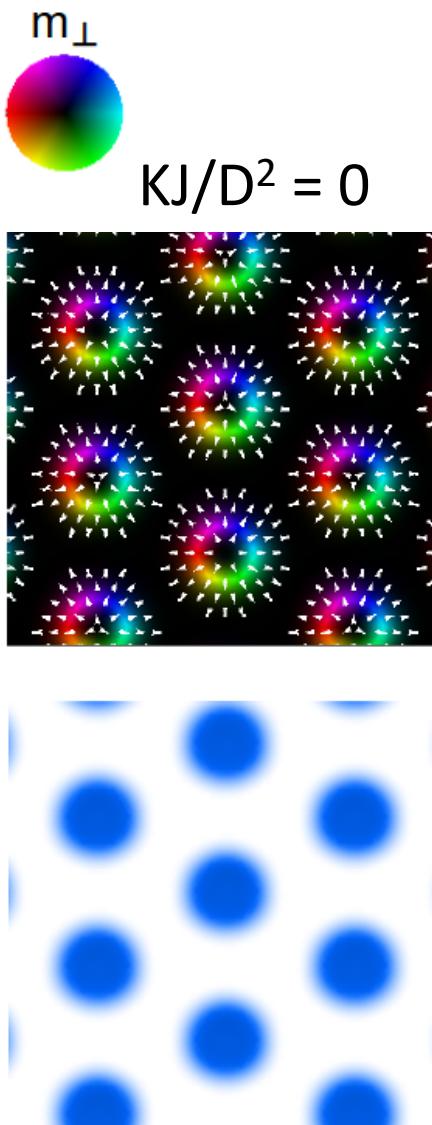
Homotopy theory

$\pi_2(S^2) = \mathbb{Z}$

Real space → Spin space

S^2

Evolution of spin-textures in 2D from $H > 2K \rightarrow H < 2K$



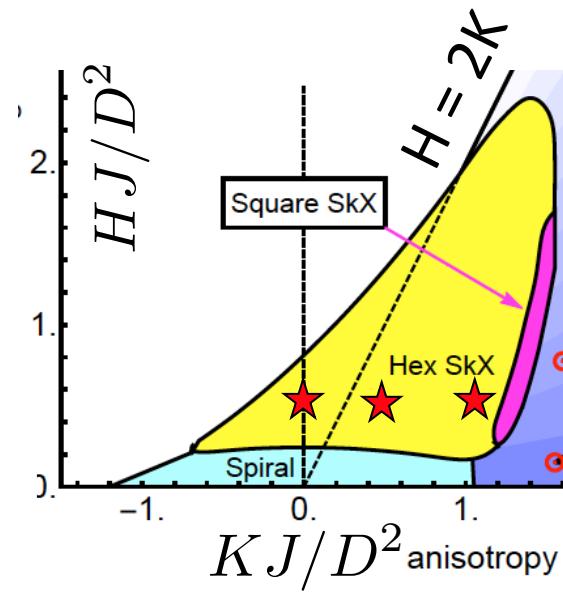
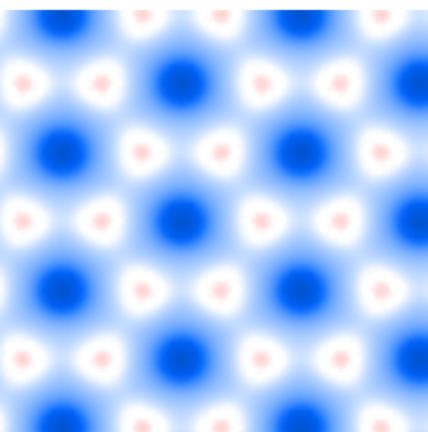
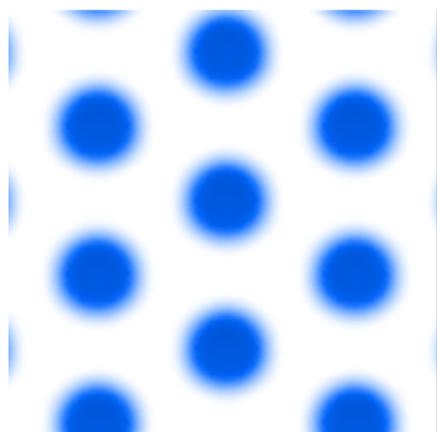
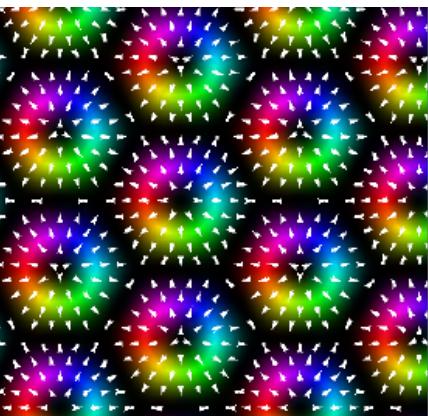
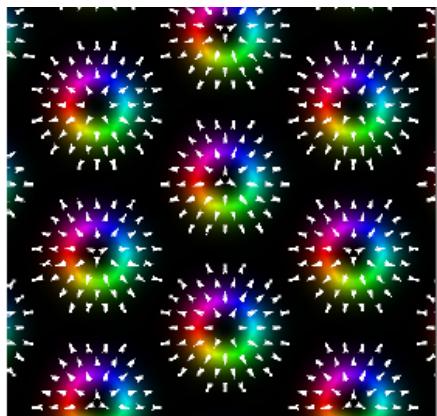
Evolution of spin-textures in 2D from $H > 2K \rightarrow H < 2K$



$$KJ/D^2 = 0$$

$$HJ/D^2 = 0.7$$

$$KJ/D^2 = 0.6$$



n_{Sk}

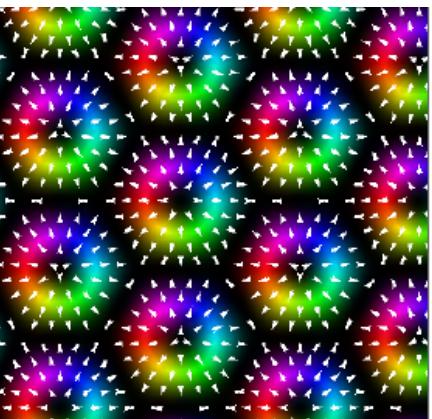
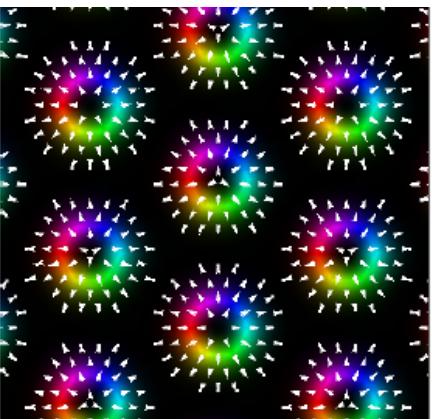


Evolution of spin-textures in 2D from $H > 2K \rightarrow H < 2K$

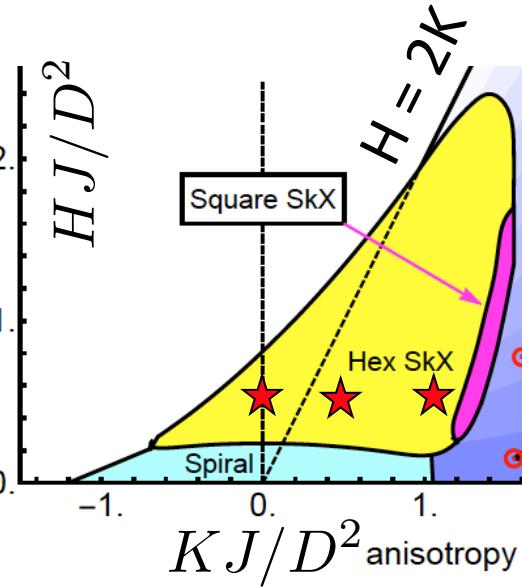
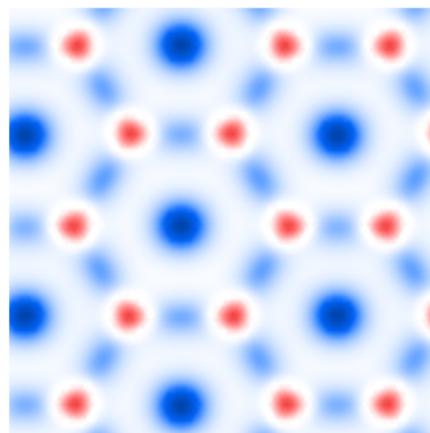
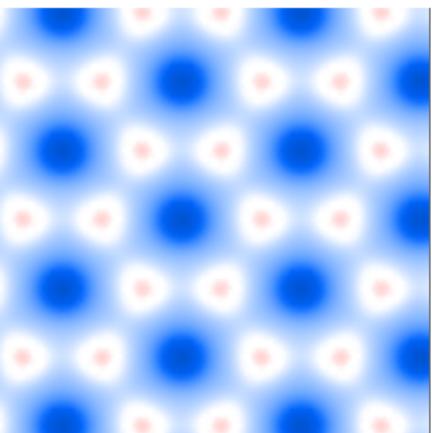
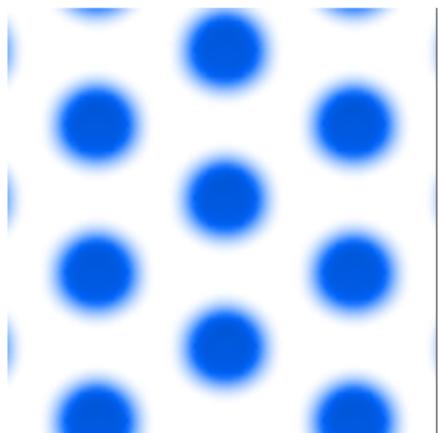
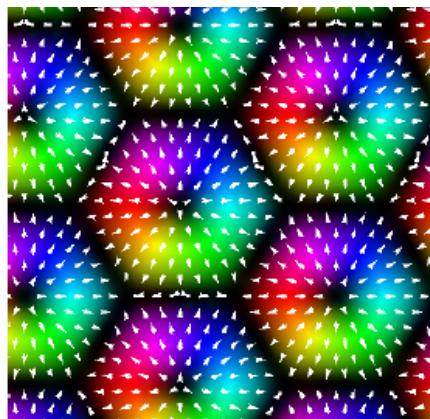


$$KJ/D^2 = 0$$

$$HJ/D^2 = 0.7$$



$$KJ/D^2 = 1.2$$



$$n_{\text{sk}}$$



Skyrmi
Density
For $H < 2K$

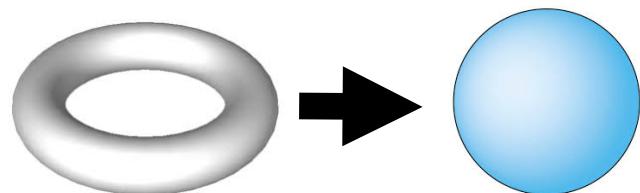
- * Not isolated at center
- * Changes sign!

Is the “topological” charge quantized for $H < 2K$?

Spins on u.c. boundary
are not all up!
Only p.b.c.’s

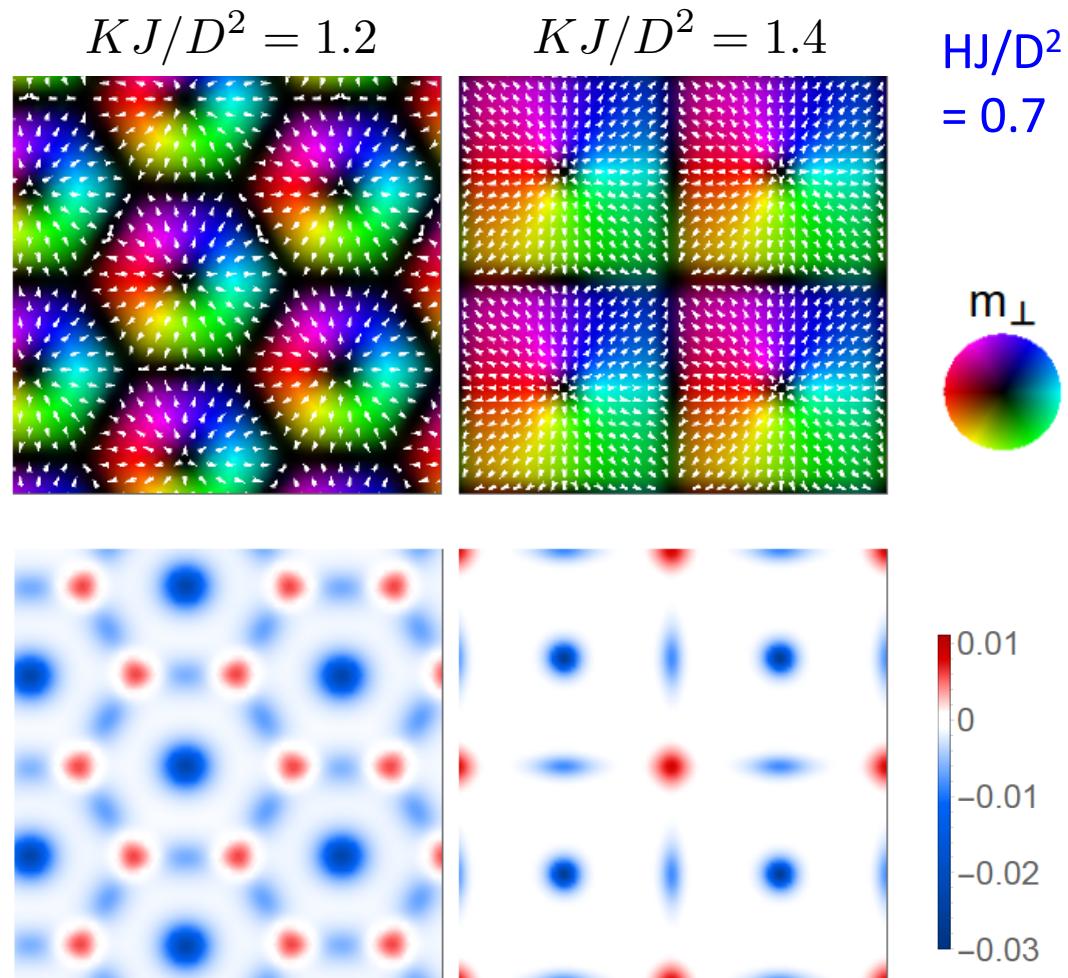
Cannot isolate texture
in a FM background

Cannot use homotopy
to get quantization



Unit cell $T^2 \rightarrow$ Spin space S^2

Chern Number
cf. Thouless et al. (TKNN)



$$n_{\text{sk}}(\mathbf{r}) = \frac{1}{4\pi} \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}})$$
$$N_{\text{sk}} = \int_{\text{unit cell}} d^2 \mathbf{r} n_{\text{sk}}(\mathbf{r})$$

$$N_{\text{sk}} \in \mathbb{Z}$$

Outline:

- Introduction
- Skyrmions in 2D vs. 3D
 - * T=0 phase diagram & role of anisotropy

- Finite temperature Monte Carlo
 - * (T,H) phase diagram in 2D

Slides of
Unpublished
Work
Omitted

- Rashba & Dresselhaus DMI
 - * Broken bulk vs. surface/mirror inversion
- Conclusions

Outline:

- Introduction
- Skyrmions in 2D vs. 3D
 - * T=0 phase diagram & role of anisotropy
- Finite temperature Monte Carlo
 - * (T,H) phase diagram in 2D
- Rashba & Dresselhaus DMI
 - * Broken bulk vs. surface/mirror inversion
- Conclusions

Symmetry \rightarrow DMI $\mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$

○ broken bulk inversion \rightarrow Dresselhaus DMI

$$\vec{\mathbf{r}} \not\propto -\vec{\mathbf{r}} \quad \rightarrow \quad \hat{\mathbf{D}}_{ij}^D = \hat{\mathbf{r}}_{ij}$$

$$\text{GL Free Energy} = -D_D \hat{\mathbf{m}} \cdot (\nabla \times \hat{\mathbf{m}})$$

○ broken surface
or mirror inversion \rightarrow Rashba DMI

$$z \not\propto -z \quad \rightarrow \quad \hat{\mathbf{D}}_{ij}^R = \hat{\mathbf{z}} \times \hat{\mathbf{r}}_{ij}$$

$$\text{GL Free Energy} = -D_R \hat{\mathbf{m}} \cdot [(\hat{\mathbf{z}} \times \nabla) \times \hat{\mathbf{m}}]$$

Systems with both Dresselhaus and Rashba DMI

Energy = FM exchange

- + Dresselhaus DMI + Rashba DMI
- + Anisotropy + Field

T = 0 Phase Diagram

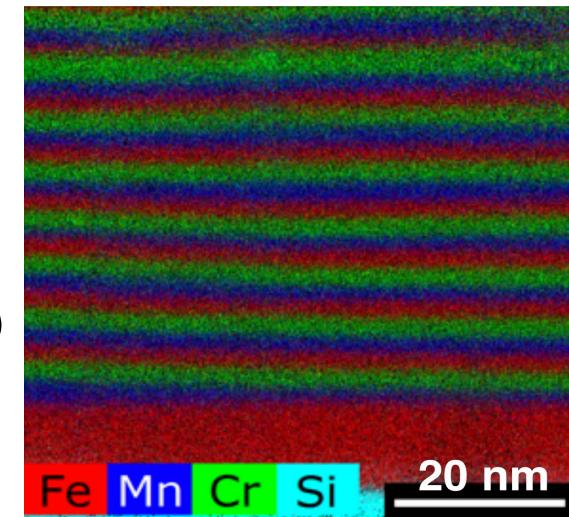
Minimize Energy → Variational & Conjugate Gradient

3D system: Bulk or sample with thickness $\gg J/D$

Materials:

- Single crystals with broken bulk inversion and mirror symmetries
- B20 Superlattices (MBE-grown CrGe/MnGe/FeG)

Ahmed, Esser, Rowland, McComb & Kawakami,
J. Crystal Growth 467, 38 (2017)



Systems with both Dresselhaus and Rashba DMI

$$D_D = D \cos \beta$$

$$D_R = D \sin \beta$$

$$D = \sqrt{D_D^2 + D_R^2}$$

$$\mathcal{F} [\hat{\mathbf{m}}(\mathbf{r})] = \frac{1}{2} |\nabla \hat{\mathbf{m}}|^2$$

FM exchange

$$+ \cos \beta \hat{\mathbf{m}} \cdot (\nabla \times \hat{\mathbf{m}})$$

DMI -- Dresselhaus

$$+ \sin \beta \hat{\mathbf{m}} \cdot [(\hat{\mathbf{z}} \times \nabla) \times \hat{\mathbf{m}}]$$

DMI -- Rashba

$$+ (AJ/D^2) m_z^2$$

Anisotropy

$$- (HJ/D^2) m_z$$

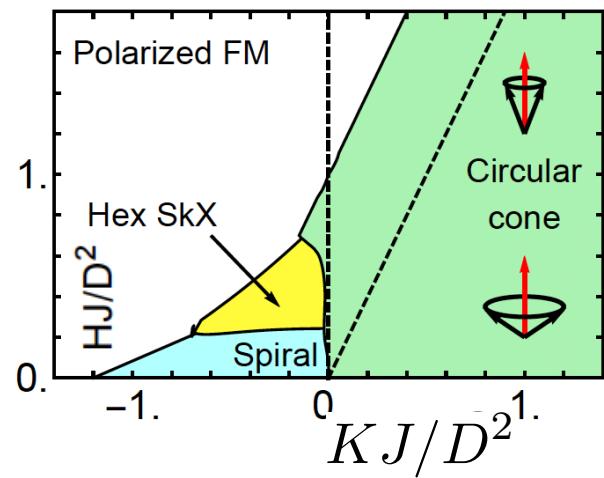
Field

$$|\hat{\mathbf{m}}(\mathbf{r})| = 1$$

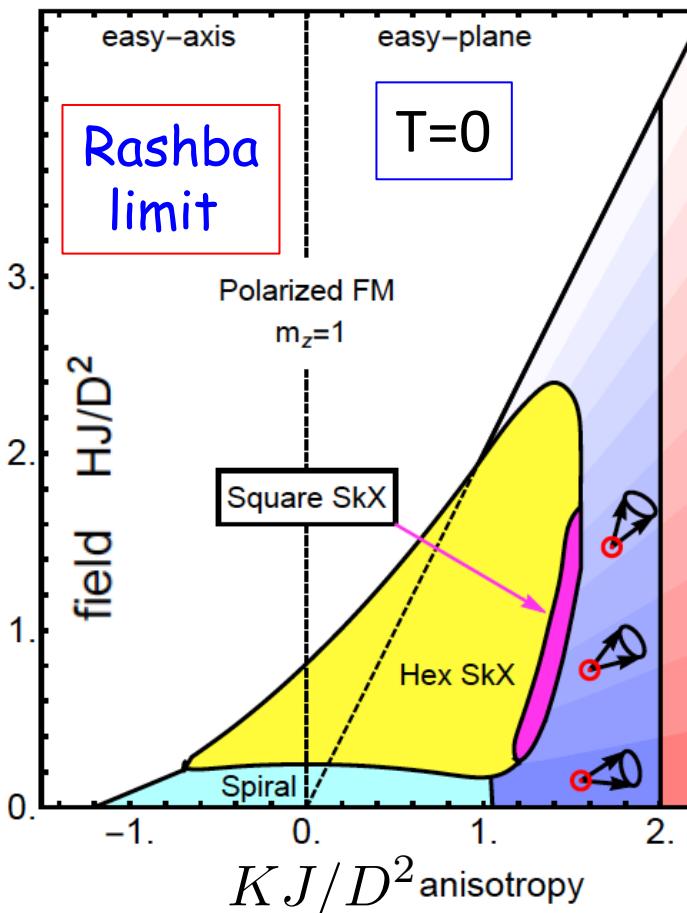
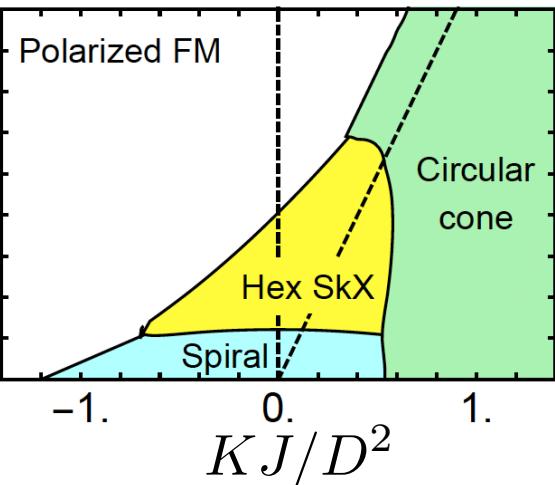
For simplicity, mirror plane normal, anisotropy axis and Field all chosen in the z-direction

Evolution from Dresselhaus DMI \rightarrow Rashba DMI

Dresselhaus limit



$D_R = D_D$



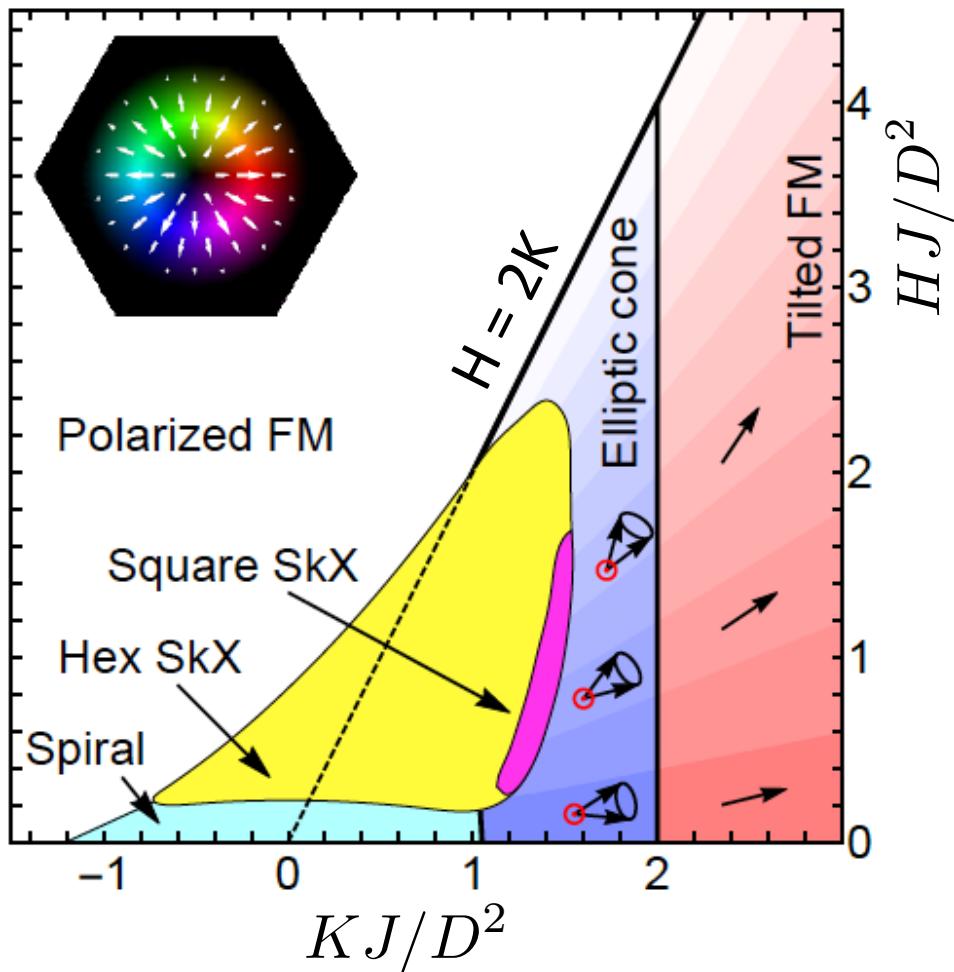
Cone: $\mathbf{m}(z)$ \rightarrow gains energy only from D_D

Spiral:

Skyrmion: $\mathbf{m}(x)$ $\left. \mathbf{m}(x, y) \right]$ \rightarrow $D = \sqrt{D_D^2 + D_R^2}$

Same Phase Diagram for

- * Rashba Limit in 3D and
- * 2D Limit



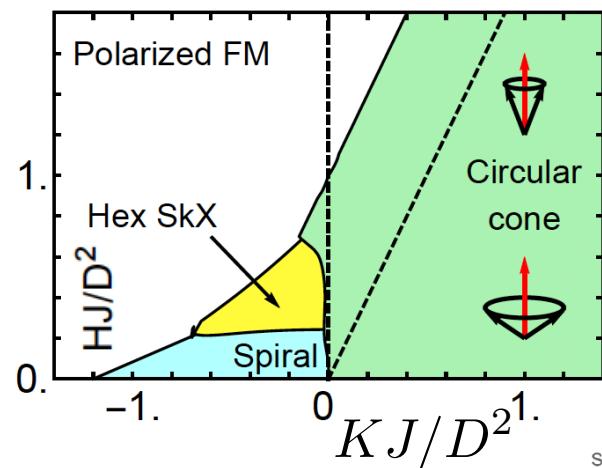
- * Rashba limit in 3D
 - No Cone Phase because no DM energy gained by z-axis “twist”
- * 2D Chiral Magnet
 - No Cone Phase because spins cannot twist along z-axis

→ Same phase diagram in both cases with spin textures that have no z-dependence, i.e.

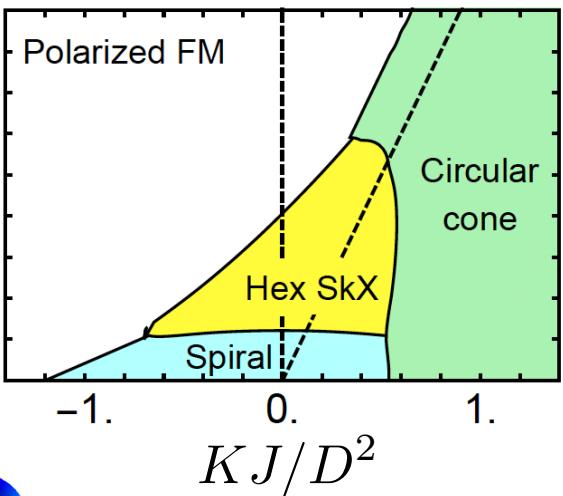
$$\hat{\mathbf{m}} = \hat{\mathbf{m}}(x, y)$$

Evolution from Dresselhaus DMI \rightarrow Rashba DMI

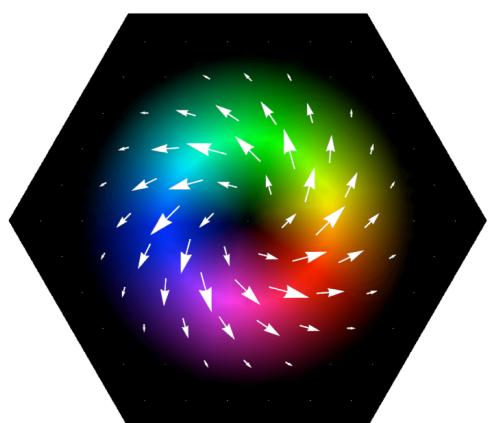
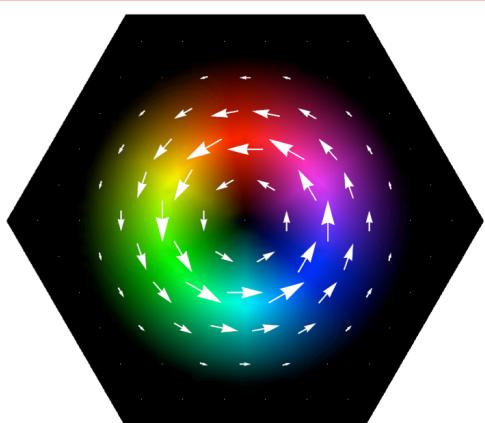
Dresselhaus limit



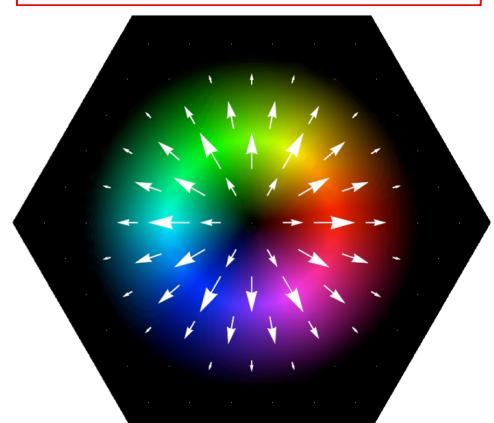
$D_R = D_D$



Vortex-like / Bloch

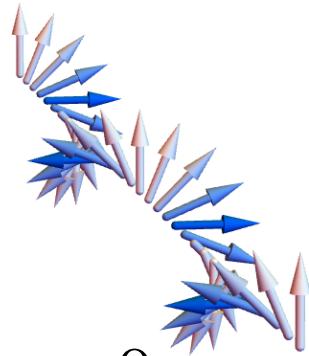


Hedgehog / Neel



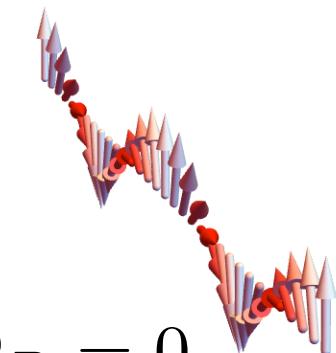
Evolution from Dresselhaus \rightarrow Rashba DMI

Helical / Bloch spiral \longrightarrow Cycloidal / Neel spiral



$$D_R = 0$$

$$\tan \beta = D_R/D_D$$

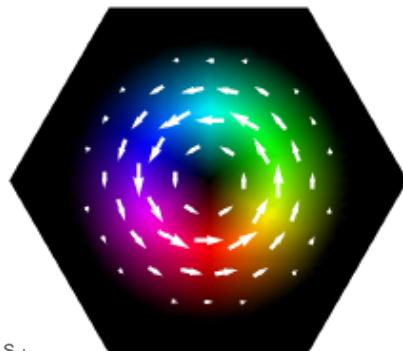


$$D_D = 0$$

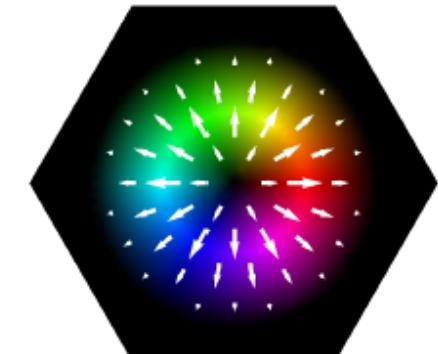
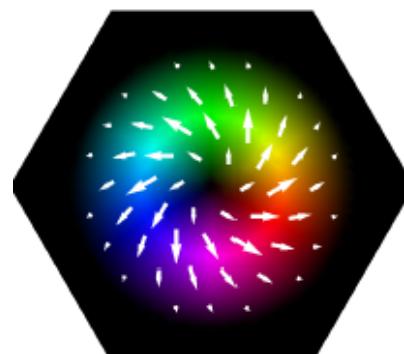
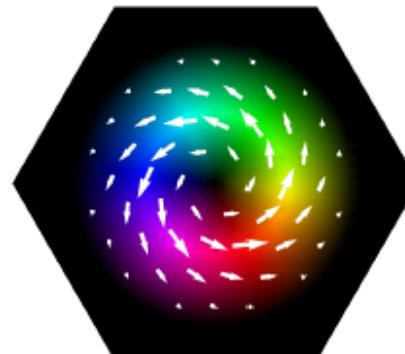
$$\hat{\mathbf{D}}_{ij}^D = \hat{\mathbf{r}}_{ij}$$

$$D_R = 0.5D_D \quad D_R = 2.0D_D$$

$$\hat{\mathbf{D}}_{ij}^R = \hat{\mathbf{z}} \times \hat{\mathbf{r}}_{ij}$$



Vortex-like
or Bloch



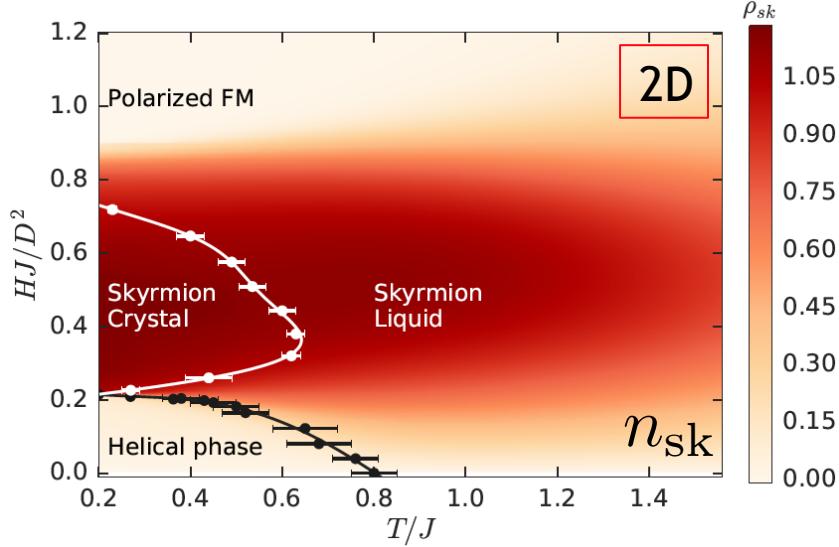
Hedgehog
or Neel

Outline:

- Introduction
- Skyrmions in 2D vs. 3D
 - * T=0 phase diagram & role of anisotropy
- Finite temperature Monte Carlo
 - * (T,H) phase diagram in 2D
- Rashba & Dresselhaus DMI
 - * Broken bulk vs. surface/mirror inversion
- Conclusions

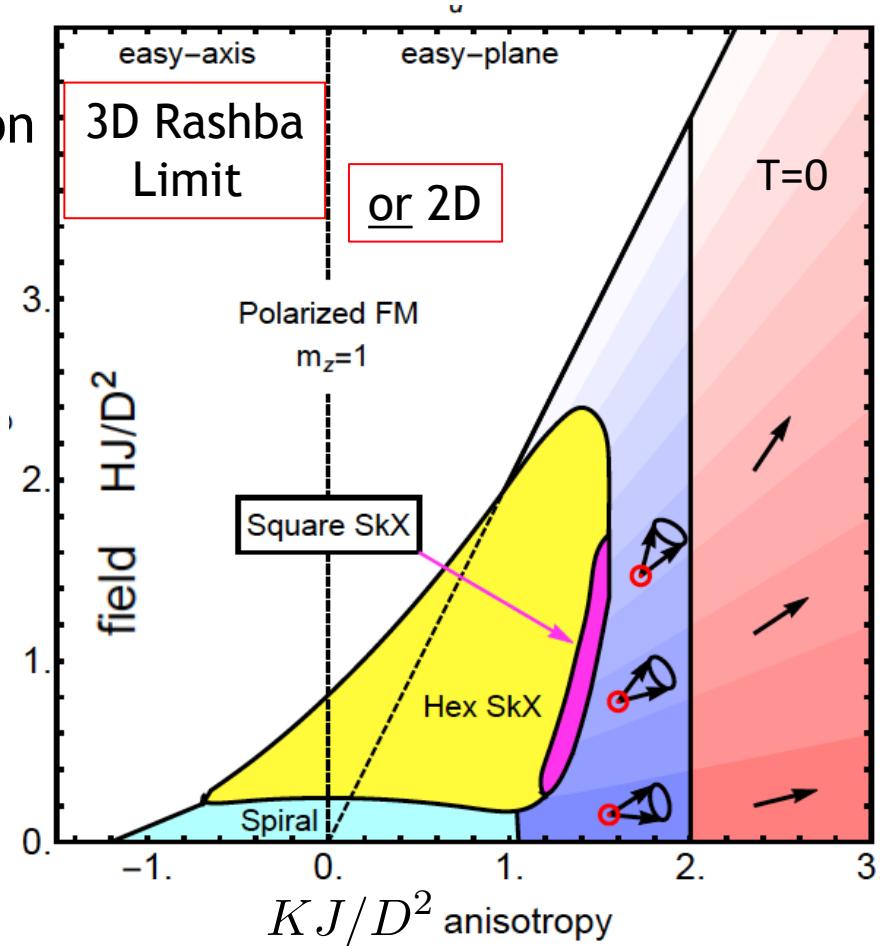
Enhanced stability of Skyrmions and Evolution of their properties

- 2D vs. 3D
- 3D
 - * broken surface/mirror inversion
 - * importance of Rashba DMI
 - * role of easy-plane anisotropy
 - * Chern number for skyrmions
- 2D (H,T) phase diagram
 - * skyrmion liquid phase



- 2D to 3D crossover with thickness → surface topological textures [arXiv:1706.08248](https://arxiv.org/abs/1706.08248)

Summary



The End